Groups and First-Order Logic

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Abstract

Some properties of groups, such as being abelian, can be expressed in first-order logic. Many others, such as being torsion, solvable, simple, or finitely generated, cannot be expressed even by a collection of first-order sentences.

Nonetheless, first-order logic can be surprisingly strong when it comes to describing groups within a reference class. A group is quasi-finitely axiomatizable if there is a single first-order sentence describing it within the class of finitely generated groups. I will discuss the impact of this notion I introduced in a 2001 paper, and its recent adaptation to profinite groups in joint work with Segal and Tent (arxiv.org/abs/1907.02262).

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