# Exact and Numerical Solutions of Nonlinear Fisher-KPP Reaction-Diffusion Equations: Design of Marine Protection Areas

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**Abstract**

Using nonclassical symmetry reduction, equations of Fisher-KPP type can be solved exactly on 2D domains subject to lethal Dirichlet boundary conditions. This leads to the critical domain size without any linear approximation.

Comparisons with fish recapture data show that diffusion (resulting from Brownian motion) is a rough approximation. The domain size can be reduced when the fish do not move randomly but are less mobile in the safe central area. Exact solutions to the nonlinear PDE can still be constructed when the diffusivity depends on location.

Another appropriate approach, validated on a recapture data set by log-likelihood improvement, is to account for non-normal displacement distributions in a two-speed mixture model to account for distinct phenotypes of station-keepers and travellers. The resulting coupled system of reaction-diffusion equations can at least be solved in the linear approximation at low population densities to find the critical domain size, even when there are disconnected patches of no-take areas. Between the safe patches, there is a negative population growth rate due to harvesting. The lethal boundary condition on no-take domains is weakened to continuity of density and continuity of flux but this does not greatly reduce the individual critical domain size. Beyond the minimal aim of species survival, advantages of multiple protected patches are (i) hedging against localised environmental disasters, (ii) promoting genetic diversity, (iii) greater spillover to the fishery by out-flux from an assured supply.

This work is in collaboration with Ashleigh Hutchinson (WITS Univ. South Africa), Bruce Mann (Univ. Kwa-Zulu Natal, South Africa) and Xia Li (La Trobe Univ.)

*Keywords:* (n/a).

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