Perspectives on Mortality Modelling

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The modelling and management of systematic mortality risk are two of the main concerns of large life assurers and pension plans:

Modelling:

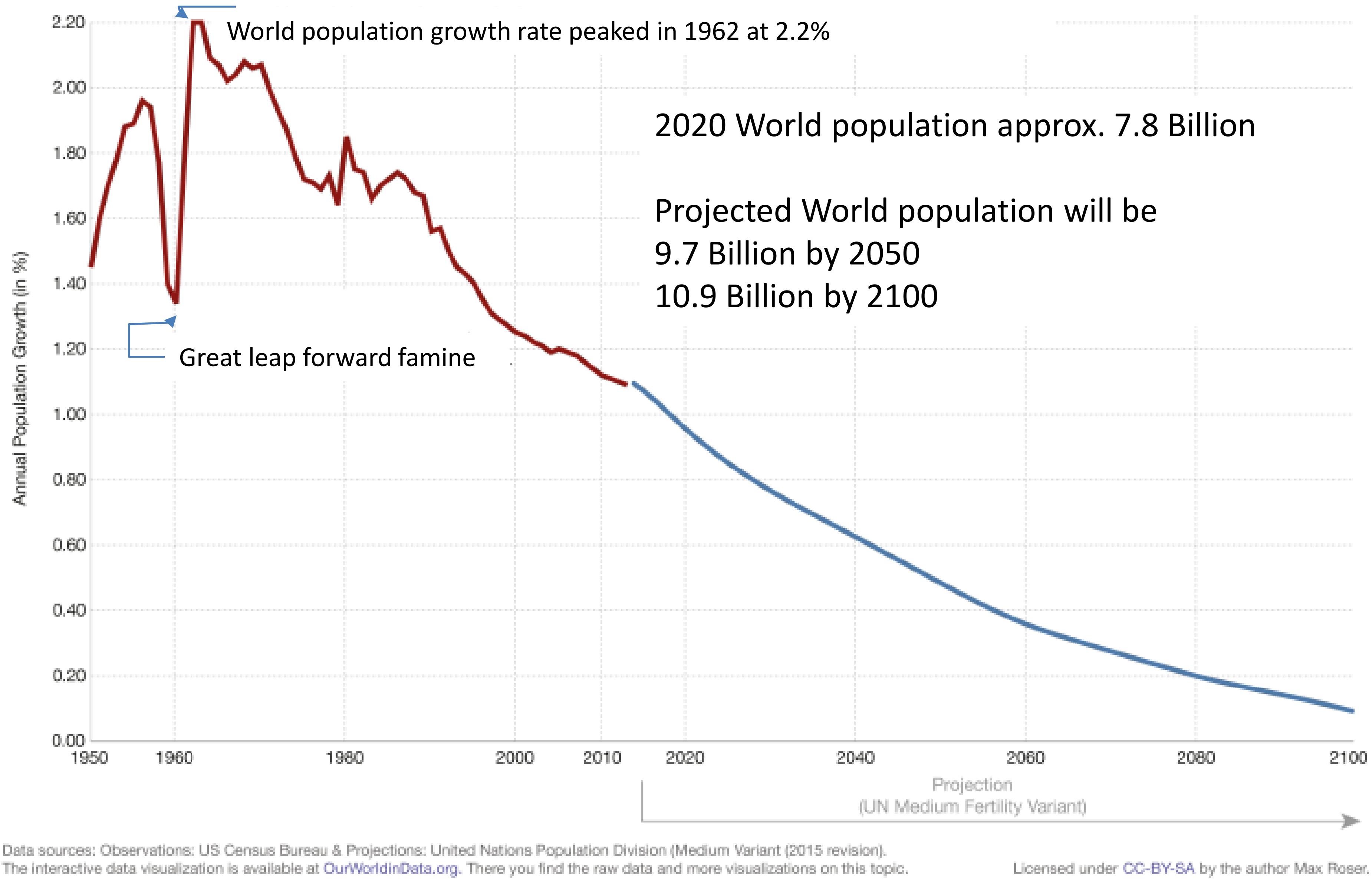
- What is the best way to forecast future mortality rates and to model the uncertainty surrounding these forecasts?
- How do we value risky future cashflows that depend on future mortality rates?

Management:

- How can this risk be actively managed and reduced as part of an overall strategy of efficient risk management?
- What hedging instruments are easier to price than others?

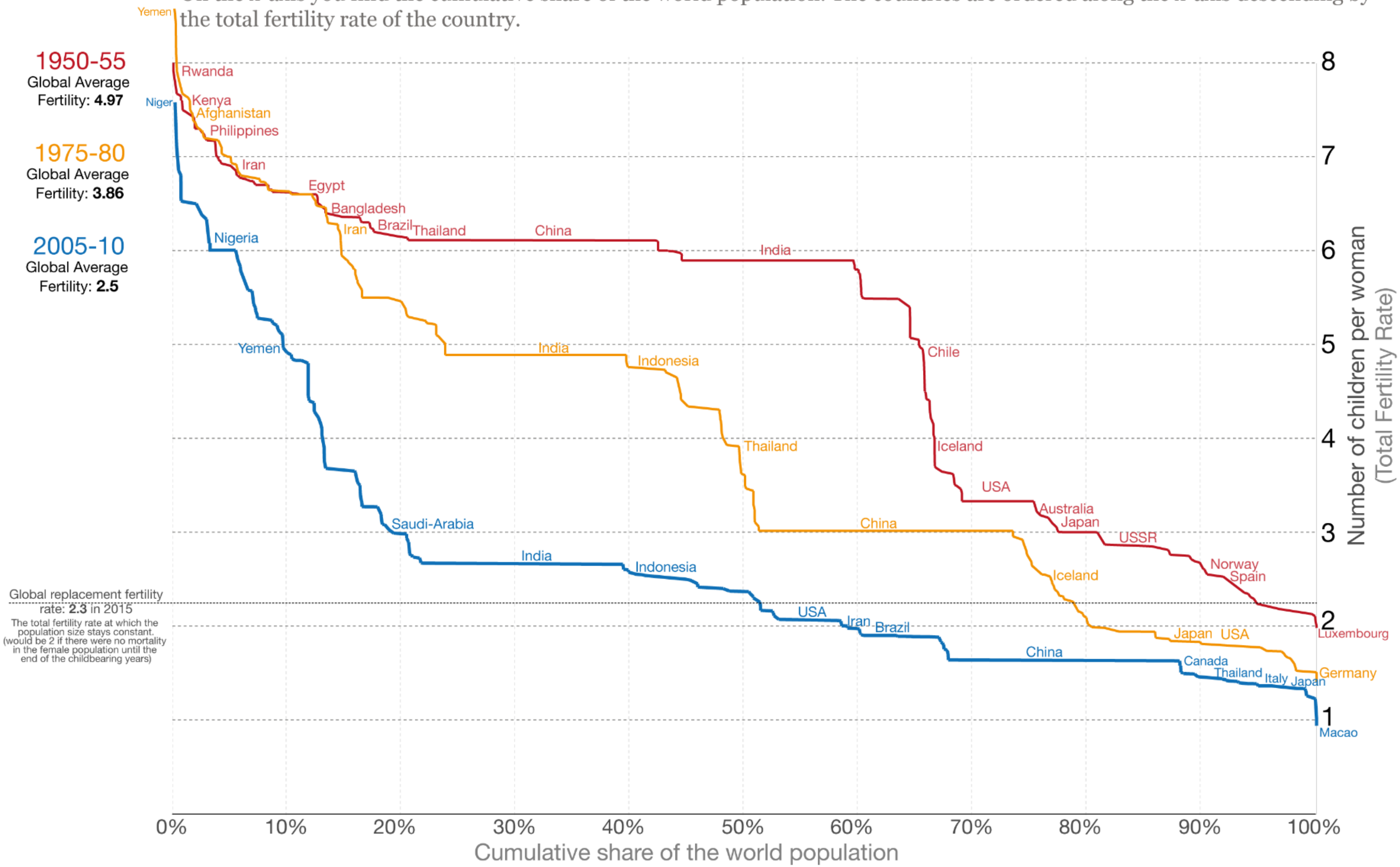
Context of Mortality Modelling and Population Demographics

Annual world population growth rate (1950-2100)



World population by level of fertility over time (1950-2010)

On the x-axis you find the cumulative share of the world population. The countries are ordered along the x-axis descending by the total fertility rate of the country.

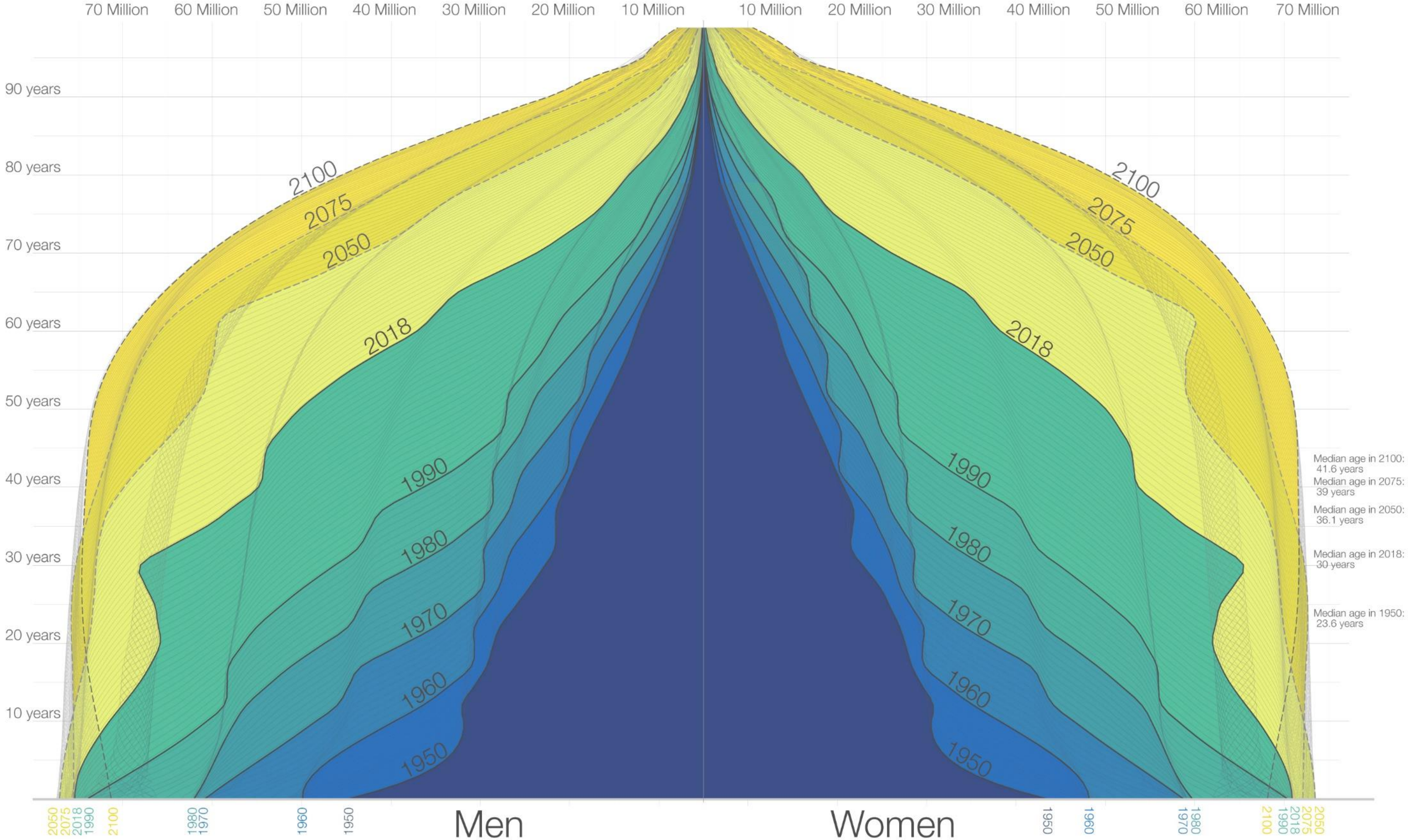


Data source: United Nations Population Division (2012 revision).

The interactive data visualization is available at OurWorldinData.org. There you find the raw data and more visualizations on this topic.

The Demography of the World Population from 1950 to 2100

Shown is the age distribution of the world population – by sex – from 1950 to 2018 and the *UN Population Division*'s projection until 2100.



Data source: United Nations Population Division – World Population Prospects 2017; Medium Variant.

The data visualization is available at OurWorldinData.org, where you find more research on how the world is changing and why.

What about Life Expectancies?

Two Components:

- **life expectancy** (the average number of years that an individual is expected to live based on current mortality rates)

- **healthy life expectancy** (the average number of years that an individual is expected to live in a state of self-assessed good or very good health)

NOTE:
Expectations of life are often calculated in one of two ways:

- Period life expectancy and cohort life expectancy.

Cohort life expectancies are calculated using age-specific mortality rates, which allow for known or projected changes in mortality in later years.

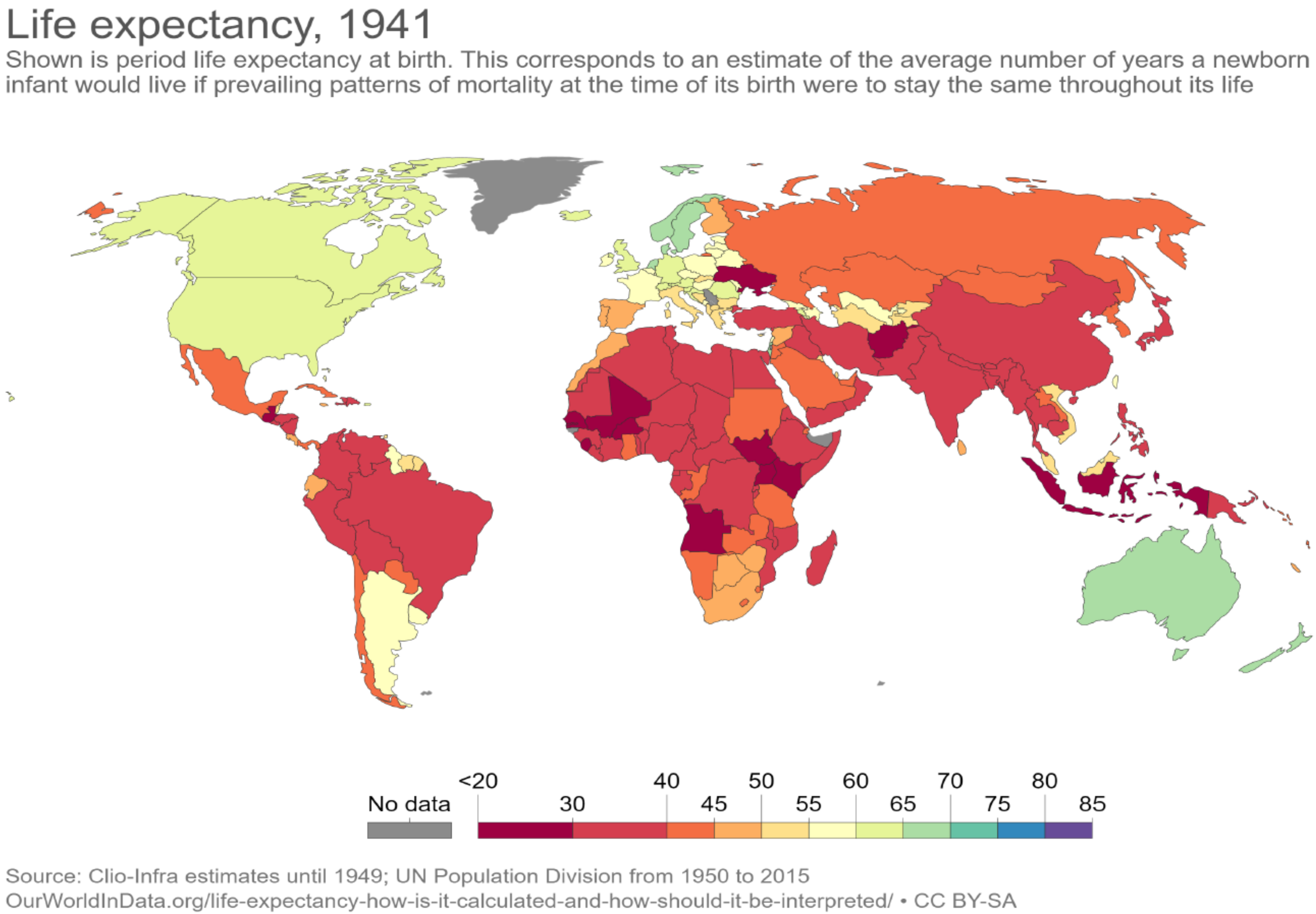


Figure: Global life expectancy by country in 1941

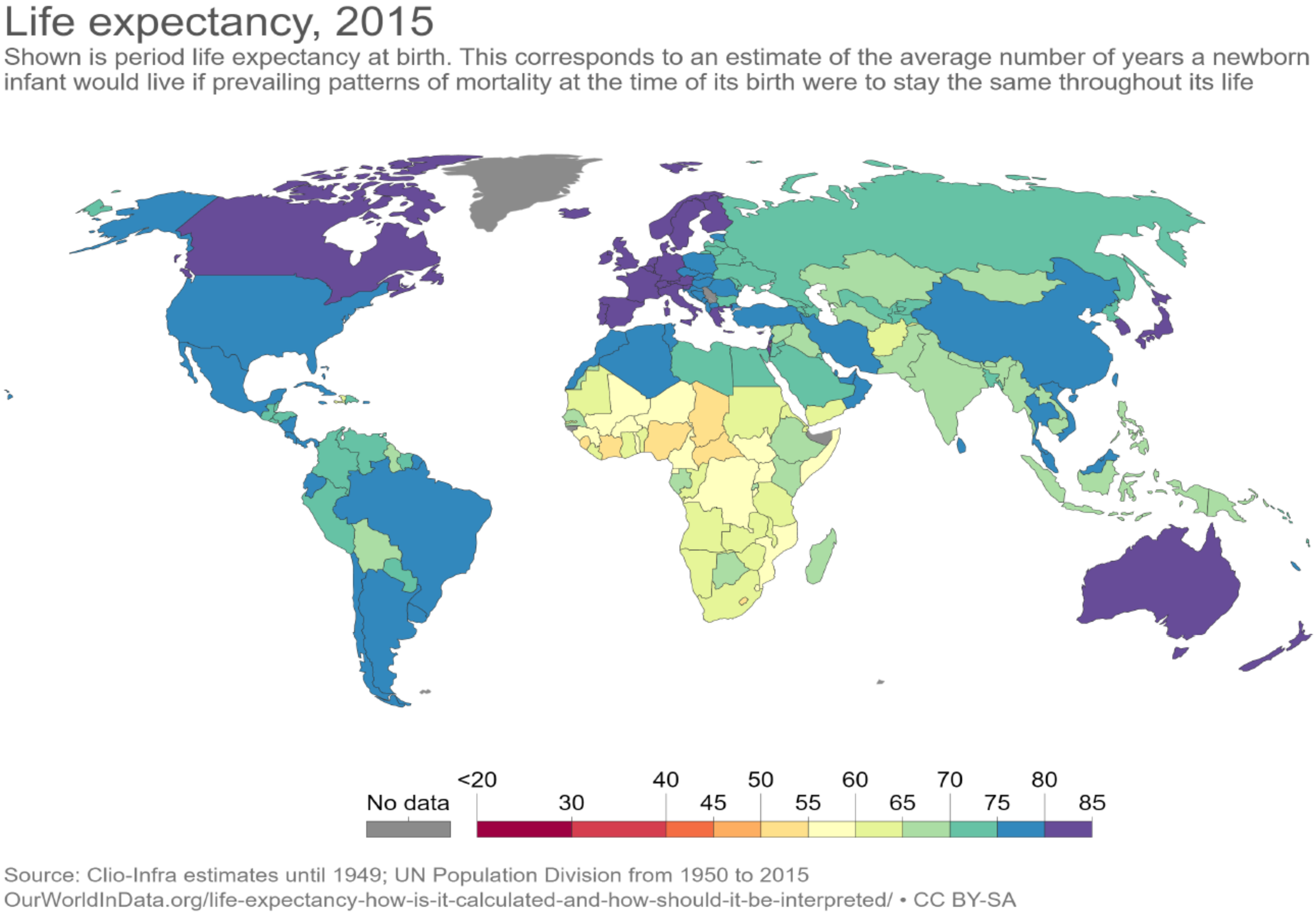
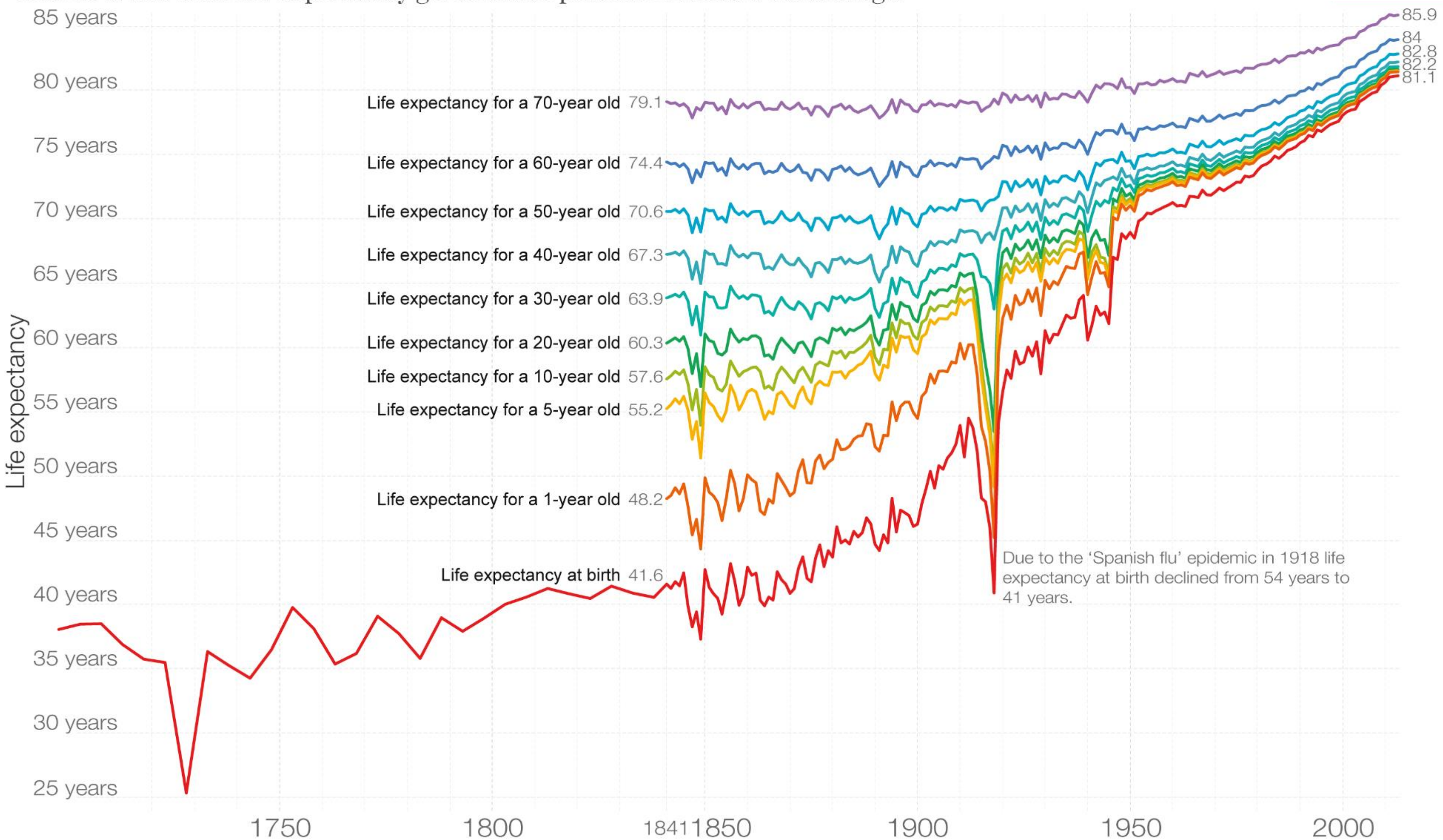


Figure: Global life expectancy by country in 2015

Life Expectancy by Age in England and Wales, 1700-2013

Shown is the total life expectancy given that a person reached a certain age.



Data source: Life expectancy at birth Clio-Infra. Data on life expectany at age 1 and older from the Human Mortality Database.

OurWorldinData.org – Research and data to make progress against the world's largest problems.

Whilst life expectancy is increasing so to is the number of disabled/unhealthy years of life
→ **unhealthy years of life where medical, care and reduced ability to work are increasing**

2013-2015 UK

At birth	**At birth**		**At age 65**	
	Males	Females	Males	Females
Life expectancy	79.5	83.1	18.7	21.1
Healthy life expectancy	**63.4**	**64.1**	10.5	11.2
Number of years in poor health	16.1	19.0	8.2	9.9
% of life in poor health	**20.3**	**22.9**	**43.9**	**46.9**

Office of National Statistics (UK government)

- Ageing populations are a major challenge for many countries.
    - Fertility rates are declining while life expectancy is increasing.

- longevity risk: the adverse financial outcome of people living longer than expected $\Rightarrow$ possibility of outliving their retirement savings.

    - long term demographic risk has significant implications for societies and manifests as a systematic risk for pension plans and annuity providers.

- Policymakers rely on mortality projection to determine appropriate pension benefits and regulations regarding retirement.

Stochastic Mortality Models

Modelling Mortality Stochastically:

A diverse range of models have been proposed in the literature for stochastic mortality modelling:

Single age group models:

- Model the individual mortality evolution either the force of mortality or the annual death counts.
- Typically such models include: temporal smoothing splines; demographic factors; can be count processes or functional regressions (or both); ARIMA type structures.

Term structure of mortality (multiple age group) models:

- Typically model the log mortality rate across the term structure of mortality.
- Typically such models include: temporal smoothing splines; period effects; cohort effects.

Evidence for Additional Structure in Time-Series Regression Models for Mortality Projection

Long Memory/Persistence

What is a Long Memory Feature?

- Long memory basically refers to the level of statistical dependence between two points in a time series.

- Given a stationary time series process $Y \equiv \{Y_t\}_{t=1:T}$, with $Y \in (\mathbb{N} \cup \{0\})^T$, [Beran, 1994] defined a condition for long memory stationary process via divergence of the autocorrelation function (ACF):

$$\lim_{n \to \infty} \sum_{j=-n}^{n} |\rho(j)| \to \infty \quad \text{where} \quad \rho(j) = \frac{\mathrm{Cov}(Y_t, Y_{t+j})}{\sqrt{\mathrm{Var}(Y_t), \mathrm{Var}(Y_{t+j})}}.$$

- Since the early work of [Hurst, 1951], long memory phenomena has been well recognized in diverse fields of application.

To understand Long Memory:

we need to think about back shift and difference operators in time series to obtain temporal Integration of a time series.

- Consider a time series $\{Y_t\}_{t=1:n}$ then the backshift operator and difference operator give

$$BY_t = Y_{t-1}$$
$$(1 - B)Y_t = Y_t - Y_{t-1}$$

- Typically one considers in an Integrated ARI(d)MA model a differencing operator $(1 - B)^d$ for integers $d \in \mathbb{N}$ which is used to "differentiate" temporal trends from the time series to make it weakly stationary.

In a long memory model one replaces integer difference operators with fractional differential operators
$\Rightarrow$ by setting d to be fractional $(0 < d < 1/2)$

Extending to fractional differences:

- The fractional difference operator has a generator, expressed in integer powers of back shift operator B:

$$(1-B)^{-d} = \sum_{j=0}^{\infty} \frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)} B^j$$

which has a lag decay much slower than exponential (with rate depending on d)

- the $\mathrm{ARFIMA}(0, d, 0)$ model describes a long memory stationary process with a hyperbolic decay of the ACF as compared to the geometric decay for an ARIMA model.

- Oscillatory autocovariance is obtained using the generalised difference operator:

$$(1 - 2\mathrm{u}\mathrm{B} + \mathrm{B}^2)^{-\mathrm{d}} = \sum_{\mathrm{j}=0}^{\infty} \Psi_{\mathrm{j}}(\mathrm{u})\mathrm{B}^{\mathrm{j}}$$

where $\Psi_{\mathrm{j}}(\mathrm{u})$ is the j-th order Gagenbauer orthogonal polynomial

$$\Psi_{\mathrm{j}}(\mathrm{u}) = \sum_{\mathrm{q}=0}^{[\mathrm{j}/2]} \frac{(-1)^{\mathrm{q}}(2\mathrm{u})^{\mathrm{j}-2\mathrm{q}}\Gamma(\mathrm{d}-\mathrm{q}+\mathrm{j})}{\mathrm{q}!(\mathrm{j}-2\mathrm{q})!\Gamma(\mathrm{d})}.$$

- This fractional parameter d captures the persistence for a given long memory model.

Relationship between long memory d and Hurst exponent H

Weak convergence of fractional time series to fBM:

Define a fractional difference characterizing process

$$Q_t = (1-B)^{-d}\varepsilon_t, \quad \varepsilon_t \overset{i.i.d}{\sim} N(0,1), \quad |d| < 1/2,$$

then the Q_t is a stationary and weakly dependent process.

Now consider the following scaled partial sum constructed from this process,

$$Z_T(\xi) := \frac{\sum_{t=1}^{[T\xi]} Q_t}{\sigma_T}, \quad 0 \le \xi \le 1,$$

where $\sigma_T^2 = \mathbb{E}\left[\left(\sum_{t=1}^{T} Q_t\right)^2\right]$ and $[\cdot]$ denotes integer part.

[Davidson, 2000] showed the following theorem

Theorem

The following weak convergence holds for all $|d| < 1/2$ and all $0 \leq \xi \leq 1$,

$$Z_T(\xi) \xrightarrow{d} V_d^{-\frac{1}{2}} W_d(\xi), \quad \text{as} \quad T \to \infty,$$

with the scale constant

$$V_d = \frac{1}{\Gamma(d+1)^2}\left(\frac{1}{2d+1} + \int_0^\infty ((1+\tau)^d - \tau^d)^2 d\tau\right),$$

chosen to ensure that $\mathbb{E}\left[W_d(1)^2\right] = 1$. For $|d| < 1/2$, $W_d(\xi)$ is a fractional Brownian motion with the following representation

$$W_d(\xi) = \frac{1}{\Gamma(d+1)}\left(\int_{-\infty}^0 \left((\xi - s)^d - (-s)^d\right) dW(s) + \int_0^\xi (\xi - s)^d dW(s)\right),$$

where W is the standard Brownian motion.

The relationship between d and H is then given by $d = H - 0.5$.

How to detect long memory features in mortality time series?

- Statistically testing directly for persistence/long memory properties in mortality time series, can be done via the Hurst exponent (denoted by H).

Developing Estimators for Long Memory Features:

- Three estimators for H that were explored included:
  - rescaled range analysis (R/S),
  - detrended fluctuation analysis (DFA), and
  - periodogram regression (PR) methods.

Rescaled Range Analysis R/S Estimators:

Given a time series $Y_{t \in (1,2,3,\cdots,T)}$, the sample mean and the standard deviation process are given by

$$\overline{Y}_T = \frac{1}{T}\sum_{j=1}^{T} Y_j \quad \text{and} \quad S_t = \sqrt{\frac{1}{t-1}\sum_{j=1}^{t}(X_j)^2},$$

where the mean adjusted series $X_t = Y_t - \overline{Y}_T$.

Then a cumulative sum series is given by $Z_t = \sum_{j=1}^{t} X_j$ and the cumulative range based on these sums is

$$R_t = \mathrm{Max}\left(0, Z_1, \cdots, Z_t\right) - \mathrm{Min}\left(0, Z_1, \cdots, Z_t\right).$$

Rescaled Range Analysis:
The following proposition proposes an estimator of H as derived in [Mandelbrot, 1975].

Theorem

Consider a time series $Y_t \in \mathbb{R}$ with std. dev. process S_t and cumulative range process R_t, then $\exists \ C \in \mathbb{R}$ such that following asymptotic property of the rescaled range R/S holds

$$[R/S](T) = \frac{1}{t}\sum_{t=1}^{T} R_t/S_t \sim CT^H, \quad \text{as} \quad T \to \infty.$$

Definition (Estimator $\hat{H}$ by R/S)

The estimator $\hat{H}$ based on the rescaled range R/S analysis is given by

$$\hat{H}_{R/S} = \frac{T(\sum_{t=1}^{T} \log R/S(t) \log t) - (\sum_{t=1}^{T} \log R/S(t))(\sum_{t=1}^{T} \log t)}{T(\sum_{t=1}^{T} (\log t)^2) - (\sum_{t=1}^{T} \log t)^2}.$$

The empirical confidence interval of $\hat{H}_{R/S}$ with sample length $T = 2^N$ [Weron, 2002] is

$$(0.5 - \exp(-7.33 \log(\log N) + 4.21), \exp(-7.20 \log(\log N) + 4.04) + 0.5),$$

We also explore various small sample adjustments and robust estimator extensions!

Long memory pattern across age groups, gender and countries

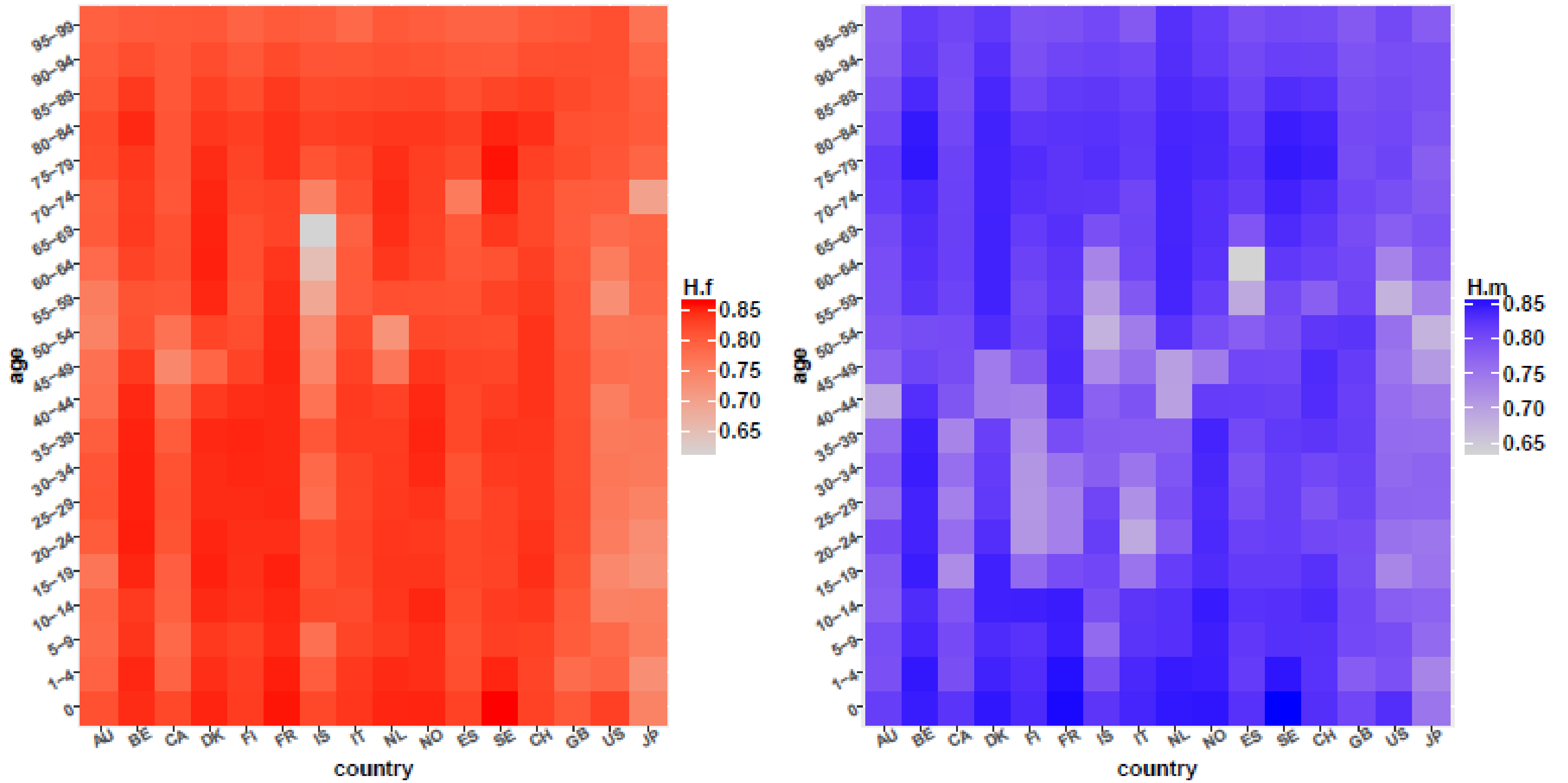


Figure: Heat map of estimated H across countries and age groups for female (left) and male (right).

All countries demonstrate some degree of long memory across all age groups.

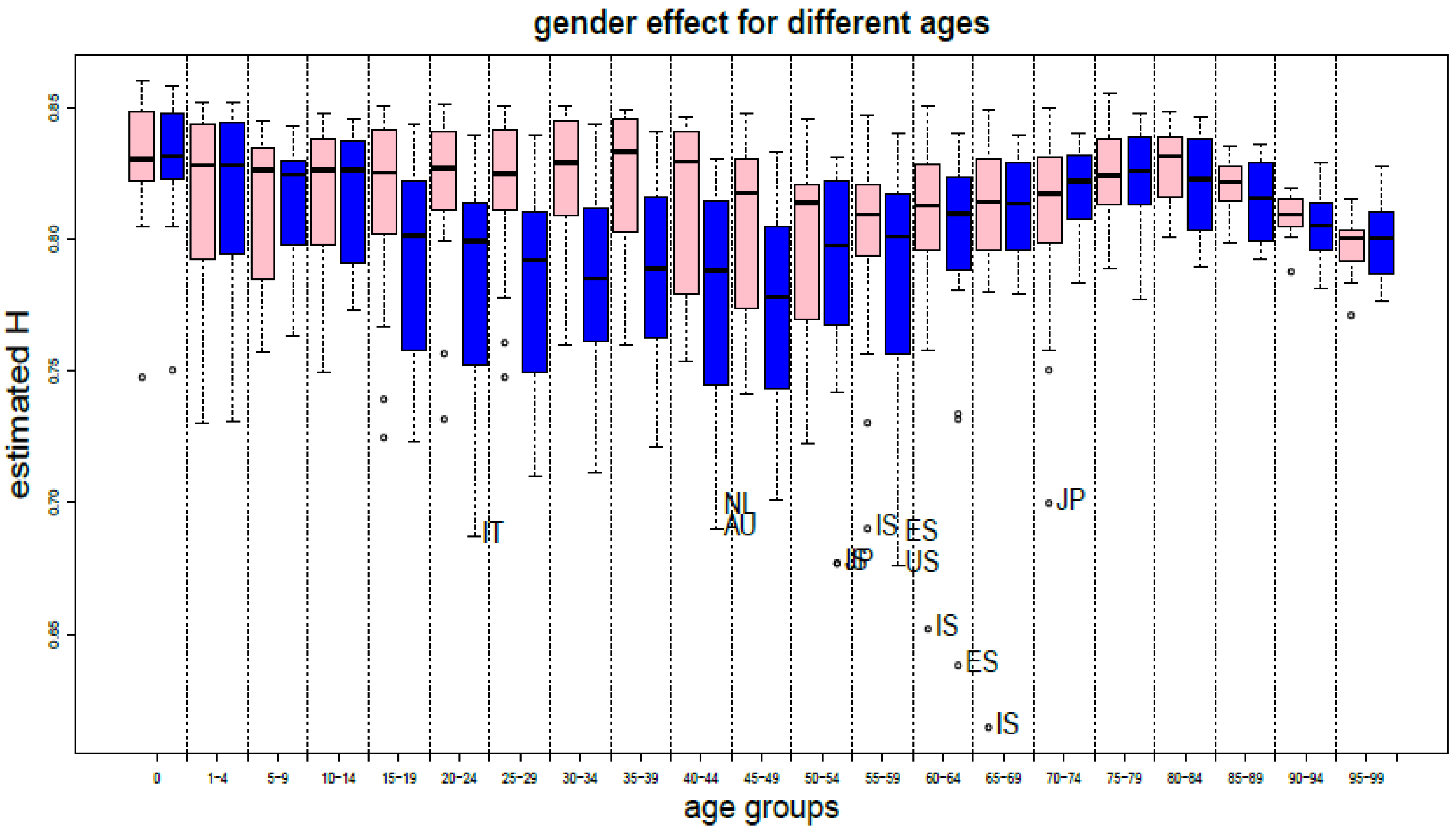


Figure: Boxplot of estimated H across age groups aggregated over countries to show gender effect. For each age group, the first boxplot is female (pink) and the second is male (blue).

Consider random vector
$Y_t = (Y_{x_1,t}, Y_{x_2,t}, \cdots, Y_{x_g,t})$ with $Y_{x,t} \in (\mathbb{N} \cup \{0\})^T$
the set of T dimensional death counts for age group
$x \in \{x_1, \cdots, x_g\}$ and years $t \in \{1, \cdots, T\}$.

Consider random vector of central exposure

$$E_t = (E_{x_1,t}, E_{x_2,t}, \cdots, E_{x_g,t})$$

Define filtrations:

$$\mathcal{F}_{1:t-1} = \sigma(Y_1, Y_2, \ldots, Y_{t-1}),$$
$$\mathcal{F}^E_{1:t-1} = \sigma(E_1, E_2, \ldots, E_{t-1}).$$

Long Memory Mortality Modelling

Two extended Bayesian models for multivariate LCC mortality models are developed:

- MELCC: no long memory with period and cohort effect;
- LMLM: long memory in trend and in cohort effect.

Definition (GLGARMA model)

Given a discrete stationary time series process $Y_{t\in\{1,2,3,\cdots,T\}}$, a GLGARMA model with order (p, d, q) is defined by

$$Y_{x,t}|\mathcal{F}_{t-1} \quad \sim \quad F(Y_{x,t};\mu_{x,t},\nu_x), \qquad (1)$$

$$(1-2uB+B^2)^d\,\Phi(B)\,(\log(\mu_{x,t})-c)=\Theta(B)\varepsilon_{x,t}, \qquad (2)$$

$$\Phi(B)=1-\phi_1B-\cdots-\phi_pB^p \ \text{ and } \ \Theta(B)=1+\theta_1B+\cdots+\theta_qB^q.$$

and F represents a discrete count distribution with parameters $\mu_{x,t}$ and ν_x which denotes the mean function and dispersion level respectively.

NOTE:

- Mean function is a Gegenbauer long memory time series
  $\Rightarrow$ a slowly damping autocorrelation function with oscillatory pattern.
- We consider a special case with $p = q = 0$ so that $\Phi(B) = \Theta(B) = 1$.

We may rewrite the trend as follows

$$(\log(\mu_{x,t})-c)=(1-2uB+B^2)^{-d}\varepsilon_{x,t}\equiv\sum_{j=0}^{\infty}\psi_j\varepsilon_{x,t-j}.$$

- Mortality models can be beneficial to incorporate both under- and over-dispersion features in distribution $Y_{x,t}|\mathcal{F}_{t-1} \sim F(Y_{x,t}; \mu_{x,t}, \nu_x)$
- Generalised Poisson (GP) distribution, [Consul, 1989] is adopted and nests Poisson as a special case.

Definition (Generalised Poisson)

Let $Y \sim GP(\mu, \nu)$ be a random variable, taking support on $\mathbb{N} \cup \{0\}$, where the pmf is given by

$$\begin{aligned} f(y; \mu, \nu) &= \mu(1-\nu)[\mu(1-\nu)+\nu y]^{y-1} e^{-\mu(1-\nu)-\nu y}/y!, \\ \mathbb{E}(Y) &= \mu \ \text{ and } \ \mathrm{Var}(Y) = \mu(1-\nu)^{-2}, \end{aligned}$$

where $\mu > 0$ is the mean parameter and $\nu \in [-1, 1)$ is the dispersion parameter. The GP distribution is over-, under- and equi-dispersed when $\nu \in (-1, 1)$ is greater than, less than and equal to 0 respectively.

Multivariate Extended Lee-Carter Cohort (MELCC) Model.

Definition (MELCC model)

$$
\begin{aligned}
&\mathrm{Y_{x,t}} | \mathcal{F}_{1:t-1}, \mathrm{E_{x,t}}, \mu_{\mathrm{x,t}}, \kappa_{\mathrm{t}}, \zeta_{\mathrm{x,t}} \sim \mathrm{GP}(\mathrm{Y_{x,t}}; \mathrm{E_{x,t}} \mu_{\mathrm{x,t}}, \nu_{\mathrm{x}}),\\
&\ln \mu_{\mathrm{x,t}} = \alpha_{\mathrm{x}} + \beta_{\mathrm{x}} \kappa_{\mathrm{t}} + \beta^{\zeta}_{\mathrm{x}} \zeta_{\mathrm{x,t}} + \varepsilon_{\mathrm{x,t}}, && \varepsilon_{\mathrm{x,t}} \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma^2_{\varepsilon}),\\
&\kappa_{\mathrm{t}} = \eta \kappa_{\mathrm{t}-1} + \varsigma^{\kappa} + \varepsilon^{\kappa}_{\mathrm{t}}, && \varepsilon^{\kappa}_{\mathrm{t}} \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma^2_{\kappa}),\\
&\zeta_{\mathrm{x_1,t}} = \lambda \zeta_{\mathrm{x_1,t-1}} + \varsigma + \varepsilon^{\zeta}_{\mathrm{t}}, && \varepsilon^{\zeta}_{\mathrm{t}} \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma^2_{\zeta}),\\
&\zeta_{\mathrm{x_i,t}} = \zeta_{\mathrm{x_{i-1},t-1}}, && \mathrm{i} = 2, 3, \cdots, \mathrm{g}
\end{aligned}
$$

- $\boldsymbol{\alpha} = [\alpha_{\mathrm{x_1}}, \cdots, \alpha_{\mathrm{x_g}}] \in \mathbb{R}^{\mathrm{g}}$ represents the profile of age groups on the log mortality rates
- $\boldsymbol{\beta} = [\beta_{\mathrm{x_1}}, \cdots, \beta_{\mathrm{x_g}}] \in \mathbb{R}^{\mathrm{g}}$ measures the interaction of age group and time effect on the log mortality rates,
- $\mathrm{E_{x,t}} \mu_{\mathrm{x,t}}$ is the mean function and the dispersion parameter $\nu_{\mathrm{x}} \in (-1, 1)$ for GP distribution.

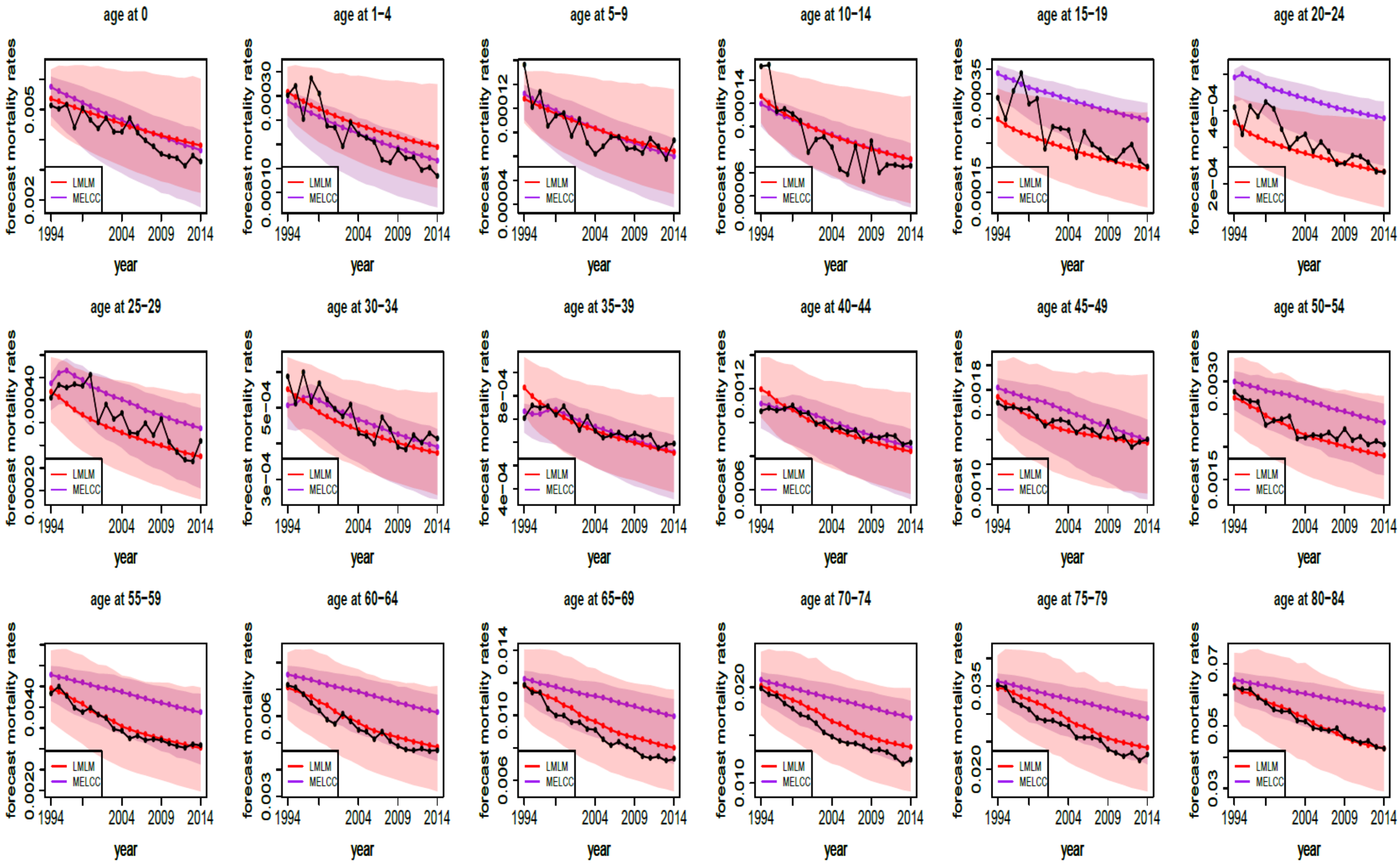
Definition (LMLM model)

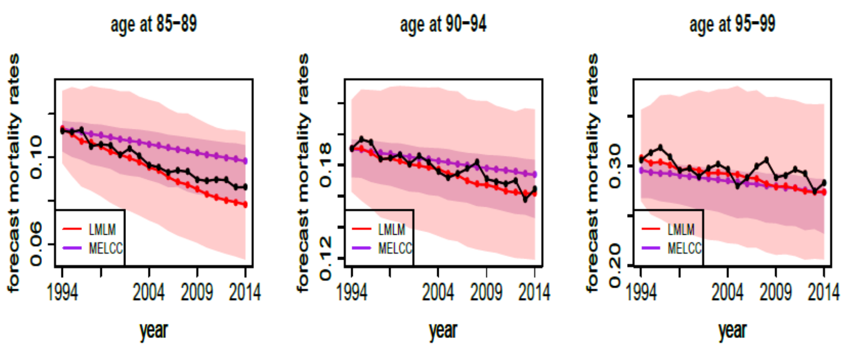
Assume $\mathrm{Y_t}|\mathcal{F}_{1:t-1}, \mathrm{E_t}, \boldsymbol{\mu}_t(\boldsymbol{\zeta}'_t)$ forms a Markov process with long memory period and cohort effect structures:

$$
\begin{aligned}
&\mathrm{Y_{x,t}}|\mathcal{F}_{1:t-1}, \mathrm{E_{x,t}}, \mu_{x,t}(\zeta'_{x,t}), \zeta'_{x,t} \sim \mathrm{GP}(\mathrm{Y_{x,t}}; \mathrm{E_{x,t}}\, \mu_{x,t}(\zeta'_{x,t}), \nu_x),\\
&\Phi_x(\mathrm{B}) \ln \mu_{x,t} = \alpha_x + \zeta'_{x,t} + \Theta_x(\mathrm{B})((1 - 2u\mathrm{B} + \mathrm{B}^2)^{-d} \varepsilon_{x,t}),\\
&\zeta'_{x_1,t} = \varsigma' + (1 - 2u'\mathrm{B} + \mathrm{B}^2)^{-d'} \varepsilon_t^{\zeta'},\\
&\zeta'_{x_i,t} = \zeta'_{x_{i-1},t-1},\ i \in \{2, 3, \cdots, g\},\\
&\qquad \varepsilon_{x,t} \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma^2_{x,\varepsilon}),\ \ \varepsilon_t^{\zeta'} \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma^2_{\zeta'})
\end{aligned}
$$

- fractional difference parameters $d \in (0, 0.5)$ and $d' \in (0, 0.5)$ control the strength of long memory
- Gegenbauer parameters u with $|u| < 1$ and u' with $|u'| < 1$ control the cycle of oscillatory autocorrelation function (ACF) for the mortality rate process and cohort effect process respectively

Example of Mortality Forecast Performance





<u>Female Australian Mortality Forecasts:</u>

- One of longest national level statistics with complex population demographics
- Known to be challenging to forecast national level statistics.

Comments on Retirement Incomes: Pensions and Defined Benefits

Insurance Sector and Government/Private Pension Preparedness:

- **Prior to 2000's:** UK, Australia,… defined benefit pension plans had limited exposure to effects of longevity risk

  → **high equity returns on pension fund wealth management portfolio's were masking impact of longevity risk**

- **Post to 2000:** **declining equity returns coupled with record low interest rate financial environments** has demonstrated the significance of decades of longevity improvements now posing a very real problem for pension schemes.

- Regulations such as Basel II and Solvency II for insurers who offer retirement income products are now also required to hold additional reserving capital to cover longevity risk.

So how well have pension providers responded to these challenges so far?

Melbourne Mercer Global Pension Index

RANK	COUNTRY	2019 INDEX SCORE
1	The Netherlands	81
2	Denmark	80.3
3	**Australia**	**75.3**
4	Finland	73.6
5	Sweden	72.3
6	Norway	71.2
7	Singapore	70.8
8	New Zealand	70.1
9	Canada	69.2
10	Chile	68.7
11	Ireland	67.3
12	Switzerland	66.7
13	Germany	66.1
14	United Kingdom	64.4
15	Hong Kong	61.9

- The average sub-index scores for 37 countries were 60.6 for adequacy, 69.7 for integrity, and 50.4 for sustainability.

Melbourne Mercer Global Pension Index

Adequacy Sub-Index (40% of a country's overall index value)
Considers how a country's pension system benefits poor and a range of income earners.

- **assesses country's household savings rate, household debt, and rate of homeownership.**

Sustainability Sub-Index (35% of a country's overall index value)
Considers sustainability of a country's retirement fund system.

- **assesses level of coverage of private pension plans, the length of expected retirement now and in the future, the labor force participation rate of older workers, government debt, and economic growth.**

Integrity Sub-Index (25% of a country's overall index value)
Considers aspects of communication, costs, governance, regulation, and protection of pension plans within that country.

- **considers the quality of the country's private sector pensions.**

Australia

(relatively well prepared)

2015 Australian Governments Intergenerational Report shows projections from the treasurer 2015 to 2055

- emphasize the stress that one of the worlds best prepared pension/superannuation schemes will face in coming decades.

- Most significant expenditure item of 2016-2017 government budget in Australia is social security and welfare (35% of tax revenue)
  → assistance for aged care is the largest component!

From 2015 to 2055:

- Number of Australians aged 65 and over will more than double
- Number of people aged between 15 and 64 for every person aged 65 and over will change from 4.5 to 2.7 people
- Age and Service Pension payments change will go from 2.9% of GDP to 3.6% of GDP
- Government expenditure on aged care services will double (change from 0.9% of GDP to 1.7%)

Conclusions

- An ageing population is a major challenge for many country in the world arising from declining fertility rate and increasing life expectancy.

- The adverse outcome of people living longer than expected, exacerbated by the record low interest rate environments, has significant implications for societies and manifests as a systematic risk for the governments and industry providers of retirement income products.

- At the same time the long term effects of defined contribution pension system (that replaced a defined benefit system) need to be better understood.

- Retirees are faced with making many complex decisions under large uncertainties of lifetime and financial returns, accounting for the government provided means-tested Age Pension (and possible policy changes) and preferences for consumption, housing and bequest.

- Furthermore, costs of health and aged care are increasing significantly making it very difficult for people and governments to plan and budget these costs due to significant uncertainty in predicting their future trends.

Research Directions in Mortality and Morbidity Modelling

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13. Murakami D. **Peters G.W.**, Matsui T. and Yamagata Y. (2020) **Spatiotemporal analysis of urban heatwaves using Tukey g-and-h random field models** (in review)

Appendix

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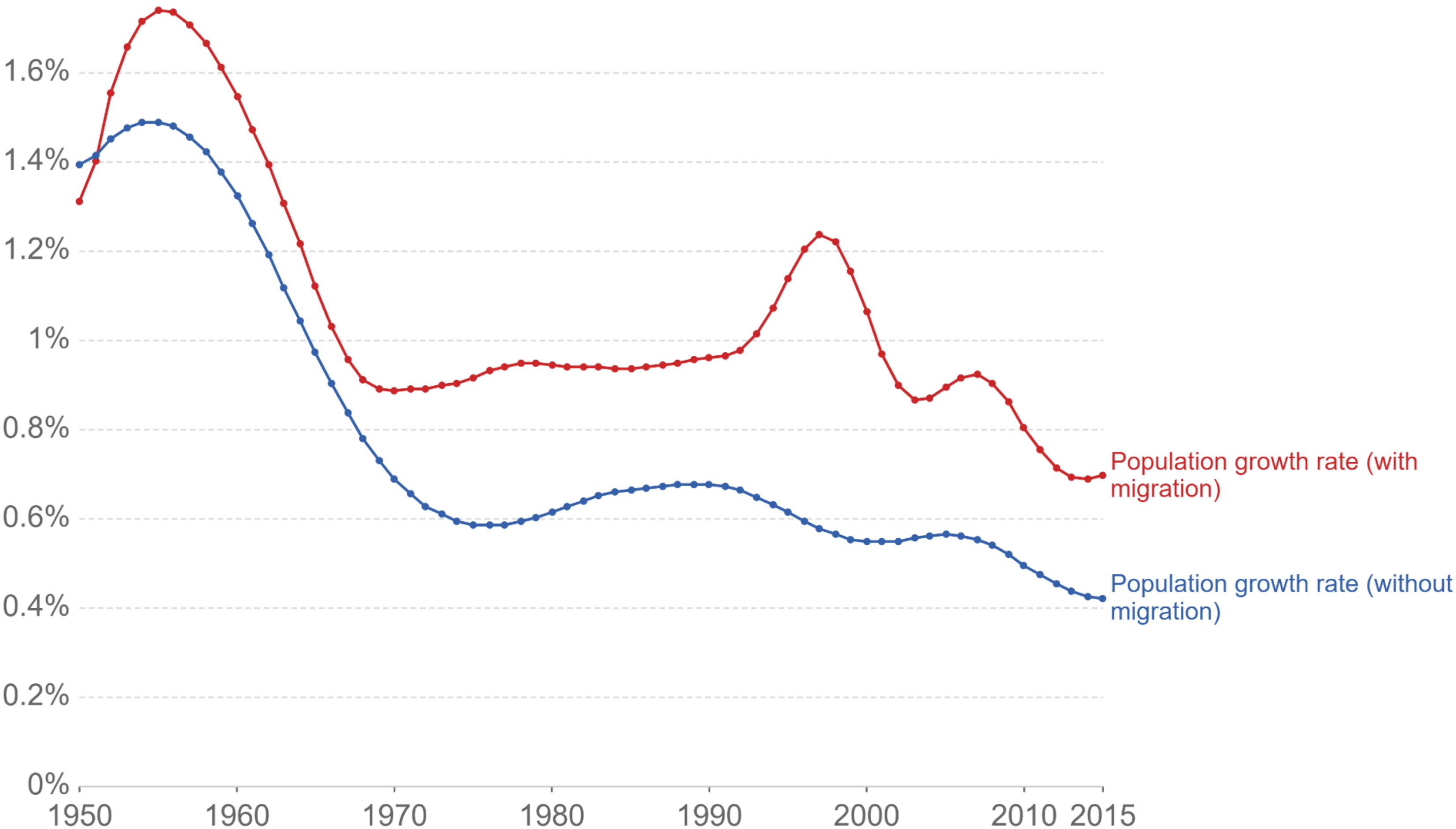
Population growth rate with and without migration, United States, 1950 to 2015

The annual change in population with migration included, versus the change if there was zero migration (neither emigration or immigration). The latter therefore represents the change in population based solely on domestic births and deaths.



Source: UN Population Division (2017 Revision)

Healthy Life Expectancy Concepts and definitions

Health state life expectancies add a quality of life dimension to estimates of life expectancy (LE) by dividing expected lifespan into time spent in different states of health or disability.

Healthy life expectancy (HLE), which estimates lifetime spent in “very good” or “good” health, is based on how individuals perceive their general health.

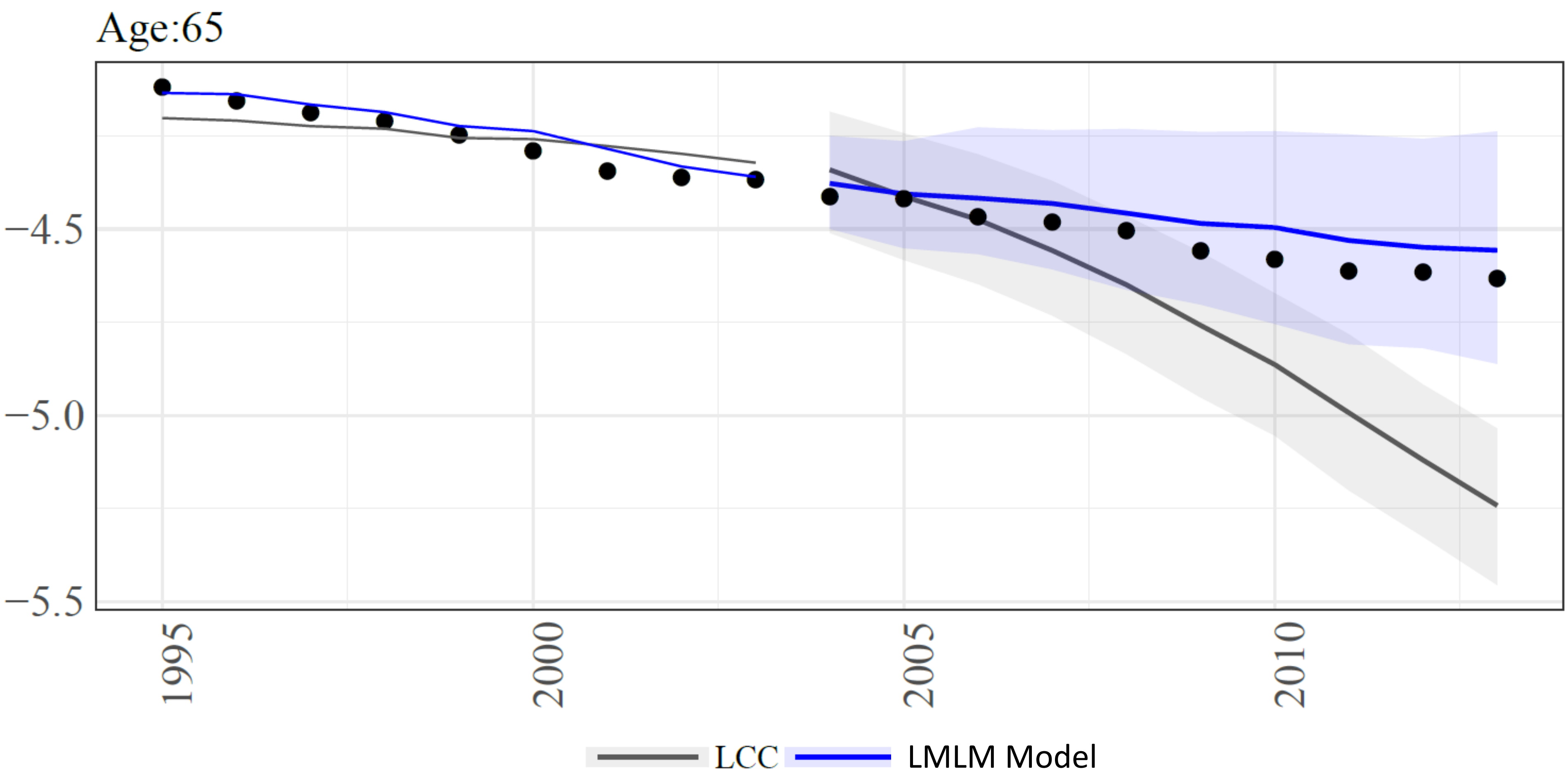
The second is disability-free life expectancy (DFLE), which estimates lifetime free from a limiting persistent illness or disability. This is based upon a self-rated assessment of how health conditions and illnesses limit an individual’s ability to carry out day-to-day activities.

DFLE is defined as the number of remaining years that an individual can expect to live without an activity restriction in carrying out normal day-to-day activities associated with a long-standing physical or mental health condition or illness.

REMARK:

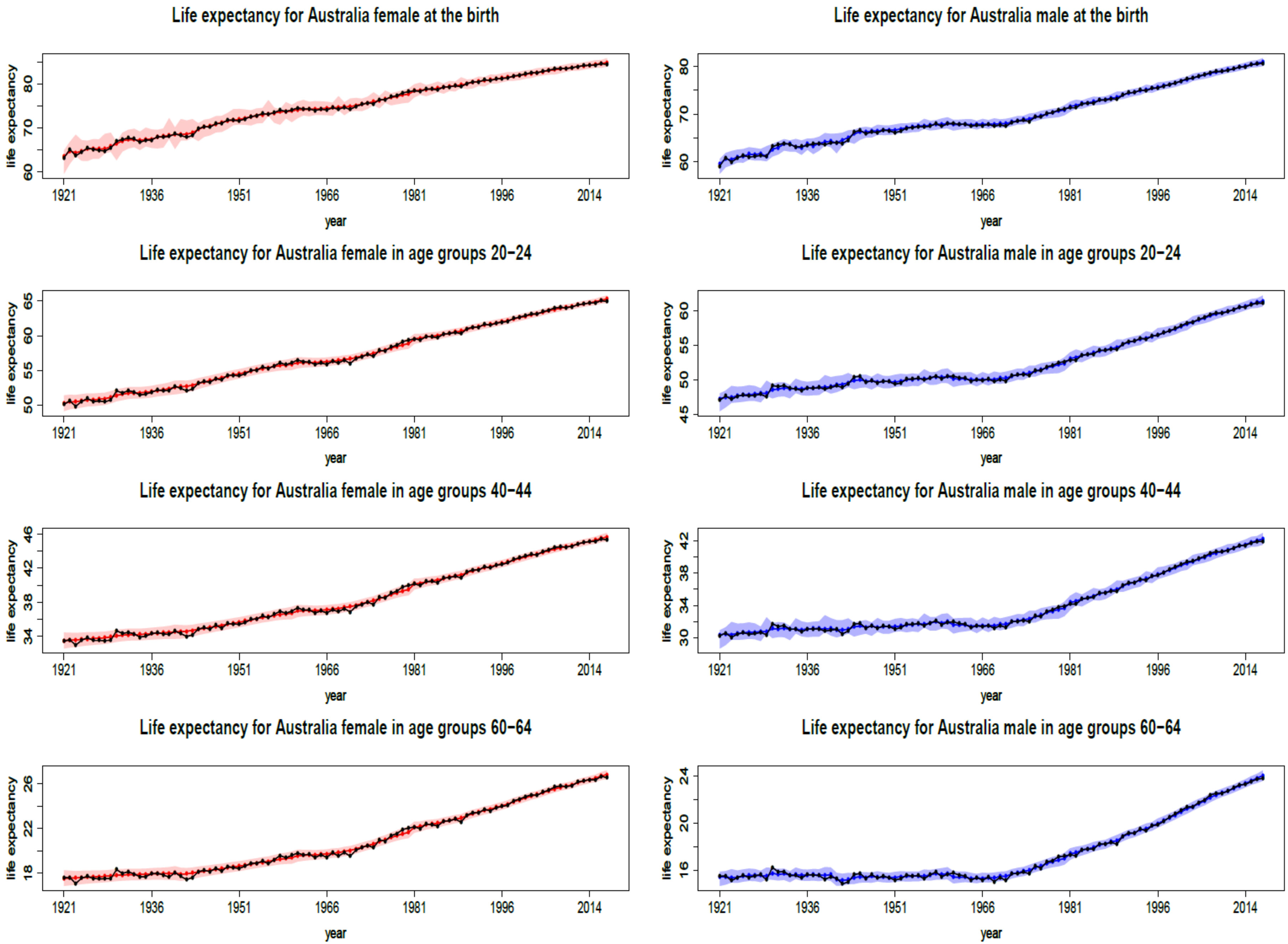
- Gegenbauer polynomial coefficients ψ_j are functionally dependent on d and u that control the strength of long memory and the oscillatory pattern respectively.
- Given the constraint $|u| < 1$, this process is stationary if $d < 1/2$, invertible if $d > -1/2$ and has long memory if $0 < d < 1/2$.
- $ARFIMA(p, d, q)$ model with mean μ is a special case of GARMA when $u = 1$.

A key example of the improvement of incorporating Long Memory into period and cohort latent intensity components improves relative to classical Lee-Carter Models.



Example of Cohort Life-Expectancy Forecast Performance

Note: blackline will be observed life expectancies but the model results are real out-of-sample forecasts which are produced in a period of time when they can be compared to the observed life-expectancies


Life expectancy for Australia female at the birth
Life expectancy for Australia male at the birth
Life expectancy for Australia female in age groups 20−24
Life expectancy for Australia male in age groups 20−24
Life expectancy for Australia female in age groups 40−44
Life expectancy for Australia male in age groups 40−44
Life expectancy for Australia female in age groups 60−64
Life expectancy for Australia male in age groups 60−64
life expectancy
year
1921
1936
1951
1966
1981
1996
2014